ECON-UA 12 — Intermediate Macroeconomics
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Problem Set 6
Due Friday, March 29

1 Labor demand in the Walrasian market

Consider the problem of a profit-maximizing firm that has Cobb-Douglas technology to produce goods. We will assume in this problem that the amount of capital is fixed at a given amount K, so that the firm only chooses labor. You can think about this problem as a problem in a very short horizon when the firm cannot adjust the amount of capital it uses (remember, capital is a stock variable, so adjustment takes time).

To study the impact of labor taxation, we will try to mimic the taxation scheme in developed countries, where both the firm and the worker pay taxes on the labor income (taxes are meant here in the broad sense, including social security and other mandatory rates).

Let us denote w the **contract wage** — i.e., the wage stated in the contract between the firm and the worker. On top of that, the firm has to pay a tax rate τ_F to the government, so that the total labor cost of the firm per one unit of labor is $(1+\tau_F)\,w$. At the same time, the worker has to pay a tax rate τ_W (this takes rate is in the interval $[0,1)$) so that the actual after-tax wage the worker receives is $(1-\tau_W)\,w$.

Question 1.1 Let us start with the problem of the firm. The profit-maximizing firm solves

$$\max_L AK^\alpha L^{1-\alpha} - (1+\tau_F)\,wL - rK.$$

Write down the first-order condition of the firm and solve for the optimal labor choice of the firm. Denote this optimal choice of labor L^D as labor **demand**.

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Question 1.2 Notice that the **labor demand curve** is the function that describes the quantity of labor demanded by the firm, L^D, as a function of the wage w.

Show (at least argue verbally) that the labor demand curve that you derived in the preceding question is a decreasing function of the wage.

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Question 1.3 Show that an increase in the tax τ_F that the firm pays will decrease the quantity of labor demanded, L^D.

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2 Labor supply in the Walrasian market

We will now turn to the labor supply by the worker. We will consider a worker who maximizes utility from consumption C and takes into account the disutility of labor L, subject to a budget constraint. The worker receives the contract wage w but he has to pay a tax rate τ_W on this wage. This means that his after-tax wage is $(1 - \tau_W)\, w$.

We can write the problem of the worker as follows:

$$\max_{C,L} \ln(C) - bL$$

subject to the budget constraint

$$C = (1 - \tau_W)\, wL + Z.$$

In the above equations, $\ln(C)$ represents the utility from consumption C, whereas $-bL$ represents the disutility of labor. The parameter b captures how much the worker dislikes labor. The second equation is the budget constraint. It tells us that the consumption C on the left-hand side has to be equal to the wealth of the worker on the right-hand side. The term $(1 - \tau_W)\, wL$ represents the total after-tax income of the worker, whereas Z represents the worker's other wealth (for instance income from non-labor activities.

Question 2.1 Use the budget constraint to substitute out consumption C in the objective function of the household. Observe that now, you have an objective function that involves only a single variable, labor L.

Take the first-order condition with respect to labor and derive the optimal choice of labor by the worker. Denote this optimal choice L^S as labor **supply**.

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Question 2.2 Notice that the **labor supply curve** is the function that describes the quantity of labor supplied by the worker, L^S, as a function of the wage w.

Show (at least argue verbally) that the labor supply curve that you derived in the preceding question is an increasing function of the wage.

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Question 2.3 Show that an increase in the tax τ_W that the firm pays will decrease the quantity of labor supplied, L^S.

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3 Equilibrium in the labor market, graphically

Although we could solve for the equilibrium in the labor market algebraically, we will proceed using a diagram. Let us start with a situation when $\tau_F = \tau_W = 0$, i.e., there are no labor income taxes.

Question 3.1 Plot a graph with wage w on the vertical axis and quantity of labor L on the horizontal axis. Into this graph, sketch the labor demand curve and the labor supply curve (decreasing or increasing depending on what you derived in the preceding exercises.

Notice that the point where the two curve intersect is the **equilibrium in the labor market**, determining the wage for which labor supply equals labor demand. Denote this equilibrium wage w^* and the equilibrium quantity of labor L^*.

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We now introduce a tax τ_W that the worker has to pay on his labor income.

Question 3.2 Which equation (labor supply / labor demand) is impacted by the introduction of the tax? How?

Hint: Use your answers from Problems 1 and 2.

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Question 3.3 Depict the impact of the tax in the graph by shifting the appropriate curve(s). Denote the new equilibrium wage w^{**} and the new equilibrium quantity of labor L^{**}.

In the same graph, also depict the after-tax equilibrium wage that the worker receives, $(1-\tau_W)\,w^{**}$.

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Observe that we said at the beginning that τ_W is the **tax paid by the worker**. But is it really? Observe that in your final graph, the pre-tax (contract) wage w^{**} is above the original equilibrium wage w^*, while the after tax wage $(1-\tau_W)\,w^{**}$ is below the original equilibrium wage w^*.

This means that imposing the tax τ_W on the worker implies that the firm has to pay him a higher pre-tax wage. Therefore, the firm also bears a share of the tax burden. When you look at the graph, you can interpret $w^{**}-w^*$ as the tax burden born by the firm, while $w^*-(1-\tau_W)\,w^{**}$ is the tax burden born by the worker.

4 Supply and demand elasticity and the burden of taxation

So far, we have analyzed the impact of taxation on the labor market equilibrium. Now we will analyze in more detail what the split of the tax burden between the firm and the worker depends on. We will see that this split depends on the elasticities of labor demand and labor supply (essentially, how steep the demand and supply curves are).

Remember that an *elasticity* is the *percentage change in one variable in response to a one-percent change in another variable.*

So, for example, the *wage elasticity of labor demand* is the percentage change in the quantity of labor demanded by firms in response to a one-percent increase in wages. Because the labor demand curve is downward sloping, the wage elasticity of labor demand should be a negative number (higher wages imply less labor demanded) but we often report the elasticity in absolute value, understanding implicitly that in the case of demand elasticities, this number is in fact negative.

When we say that the labor demand *elastic*, we mean that there is a large *decrease* in the quantity of labor demanded in response to a one-percent increase in wages. The labor demand curve is close to being flat. On the other hand, we say that the labor demand is *inelastic* when there is only a small decrease in the quantity of labor demanded in response to a one-percent increase in wages. The labor demand curve is very steep.

The same holds for the *wage elasticity of labor supply*, the only difference is that now the curve is increasing rather than decreasing. An elastic labor supply is depicted as a flat increasing curve, an inelastic labor supply as a steep increasing curve.

We often abbreviate the term 'wage elasticity of labor demand / supply' to just 'demand / supply elasticity' when it is clear that we talk about the labor market.

We can similarly define the price elasticity of demand for goods and price elasticity of supply of goods, and other elasticities, depending on the particular economic problem.

Question 4.1 Plot the standard supply-demand diagram for the labor market, with an increasing labor supply curve and a decreasing labor demand curve. Denote the equilibrium wage w^*. Now assume that the firm has to pay a proportional tax τ on the wage income.

1. Shift the labor demand curve appropriately and denote the new pre-tax equilibrium wage $(1+\tau)\,w^{**}$ and post-tax equilibrium wage w^{**}, with the new equilibrium quantity of labor L^{**}.
2. Show the tax revenue the government collects and the deadweight loss of taxation.
3. Show how the tax is split between the firm and the worker.

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Question 4.2 Assume that the labor supply is perfectly elastic (there is a given wage at which the workers are willing to supply any amount of labor, and the labor supply curve is flat). Redo the graph from the previous question.

1. Who bears the burden of paying the tax now? Why?
2. Is the deadweight loss of taxation larger or smaller than in question 4.1? Whose deadweight loss is it, the worker's or the firm's?
3. Is the change in employment larger or smaller than in question 4.1?
4. Describe situations when the labor supply is likely to be elastic.

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Question 4.3 Assume that the labor supply is perfectly inelastic (workers are willing to supply a given quantity of labor at any wage they receive). Redo the graph from the previous question.

1. Who bears the burden of paying the tax now? Why?
2. Is the deadweight loss of taxation larger or smaller than in question 4.1?
3. Is the change in employment larger or smaller than in question 4.1?
4. Describe situations when the labor supply is likely to be inelastic.

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Question 4.4 Comparing your answers from questions 4.2 and 4.3, other things equal, is it more efficient for the government to tax markets where the supply curve is elastic or inelastic?

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5 Bathtub model of unemployment and transition dynamics

Consider the bathtub model of unemployment discussed in class, described by two equations. Labor force $\bar{L}$ is divided into employed and unemployed people

$$\bar{L} = E_t + U_t$$

and the inflow into unemployment is

$$\Delta U_{t+1} = \bar{s} E_t - \bar{f} U_t. \tag{1}$$

Question 5.1 What do the parameters $\bar{s}$ and $\bar{f}$ capture?

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Question 5.2 Consider a government policy that severely limits the ability of firms to fire workers — for instance, by instituting that the firm has to pay very high severance payments to the fired worker. Which parameter of the model changes? Show that, other things equal, this decreases the steady state level of unemployment in our bathtub model.

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Question 5.3 Is this a plausible result? What happens if, if response to this policy, firms become more reluctant to hire new workers because they are worried that they will not be able to fire the worker if he turns out to be unproductive or economic conditions worsen? Which parameter would then change and what impact this would have on the steady state level of unemployment?

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Question 5.4 What do your answers to the preceding two questions imply about the usefulness of the parameters $\bar{s}$ and $\bar{f}$ for conducting policy experiments?

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Question 5.5 Suggest some government policies that may decrease the parameter $\bar{s}$ without decreasing the parameter $\bar{f}$.

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Question 5.6 Notice that you can rewrite equation (1) as

$$U_{t+1} - U_t = \bar{s} E_t - \bar{f} U_t = \bar{s}\,(L - U_t) - \bar{f} U_t \tag{2}$$

and thus

$$U_{t+1} = \left(1 - \bar{s} - \bar{f}\right) U_t + \bar{s}L \tag{3}$$

Assume that the economy starts in the steady state and the parameter $\bar{s}$ decreases. What happens to the steady state level of unemployment? By analyzing the equation above, sketch the transition path (in a graph with time on the horizontal axis) for the transition from the old to the new steady state.

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